Matrix methods for risk management
– Associated Matrices Theory

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1. Introduction

Safety is one of the fundamental human needs [4]. In the modern world natural hazards interweave with civilisation ones. Striving to improve the standard of living, supporting people’s activities with the help of machines and technology, led to a situation, in which increasing safety awareness in some areas of our lives causes its downsizing in the others. This phenomenon can be simply explained by risk transfer but understanding the problem is not equal to solving it.

Security management process is complex in terms of a multi-aspect area which is subject to action and coordination. Each decision made to increase the security is determined by the obvious economic factors, as well as by other ones of equal importance, such as social, political and media factors. This is why security management can be
described with a use of quantitative methods as extremum (minimum) seeking in the multidimensional space of expected loss, where each dimension is another aspect of the decision. Due to the fact that some of these aspects are not measurable with use of objective methods, most of the complex problems (and such are almost all emergency and crisis situations) are possible to resolve only approximately, on the basis of knowledge and experience of experts dealing with security issues.

In risk analysis theory a concept of social turbulence – a subjective component which indicates the acceptance level of risk associated with a specific hazard – has been introduced to map risk aspects which are not measurable with help of the quantitative method [7]. Taking this component into consideration enables to determine the total level of risk (including so called expert risk). Assuming that risk is the measure of security, it becomes clear that risk management is dependent on many subjective aspects and taking them into account involves using many various organisational and decision making methods.

2. Decision-making process in crisis situations

It is very difficult to describe the real decision-making process with use of the quantitative method. While one can specify the number of decision rules which may support decision making process by selecting the best alternative, from the point of view of chosen criteria, inasmuch a complete mathematical description of a decision making process is practically impossible. It is particularly difficult to describe a decision making process in complex, multiaspect situations, which are specific for crisis situations.

A factor which influences, among the others, the difficulty to describe a real decision making process is the fact, that a person who makes decisions in real situations very often needs to act under conditions of lack of information (or chaos) and deficit of time, necessary for detailed analysis or preparation.

Finally, multi aspect of real situations, especially crisis ones, when decisions made are influenced by economic, political, social, ecological and media factors, makes it impossible to concretise a simulation decision-making, hinders evaluation and consequences (effects) forecast.
Given the circumstances described above, crisis management process is cyclical and continuous, and its main aim is to analyse an occurred event and make possibly the most rational and effective decisions to ensure public safety. There are four phases of crisis management which are cyclic and occur one after another: prevention, preparation, response and recovery [11]. One of the few engineering methods for supporting a decision-making process is risk analysis\(^1\).

### 3. Risk minimising strategies

Risk minimising strategies may be divided into two categories: response and prevention (protection). We deal with an area of response when an adverse event occurs. The area of prevention is responsible for risk minimising by reduction in frequency (probability) of adverse events occurring or by minimising expected loss when the event occurred (we think of all the activities of this type taken before the event occurred). Risk analysis and estimation is to indicate areas where it is necessary to reduce risk to the value which is acceptable (admissible).

Picture 1 shows a universal risk management scheme, independent of a method used for risk level determination. As the scheme shows, it is a continuous process, very sensitive to current external conditions. When the risk exceeds the threshold of acceptability, it is necessary to plan and carry on activities aimed at risk reduction. Such activities should be conducted very cautiously, due to the fact that it is very easy to undermine the existing balance of risks. Upon identification of the problem and identifying ways to resolve it (risk reduction), it is worth considering whether putting activities into action will cause imbalance or not. However, this subject goes beyond the scope of this article.

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\(^1\) Risk can be defined as the combination of the probability of an event and its consequences. From the engineering point of view, risk can be defined as computational by conjunction of probability and effects.

*Risk Analysis* – „Using available information on sources of risk in order to estimate the risk for individuals or groups (population), property, environment or other values protected by the local community. Risk analysis usually consists of three elements: set of definitions, identification of threats and set of data necessary for risk assessment. The latter include, inter alia, sets of possible sub-scenarios which, which determine the kinetic impact of possible threats and frequency characteristics.” (from J. Wolanin, *The Basic Theory of Public Safety* 2005; translation from polish by MMS)
4. Risk analysis with the use of Risk Matrix Method

One of very popular and widely used risk analysis methods is Risk Matrix Method [2]. In the method, two basic factors affecting the value of estimated risk are taken into consideration: probability of an adverse event (pre-determined scenario of events) and its consequences. Gradation of probability of event occurrence is done in a conventional scale (usually of 5 tiers): from 1 – the least likely event, to 5 – highly possible event, and their effects: from A – minor effects (very little), to E – huge effects, disastrous. Picture 2 shows a pattern risk matrix.
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Fig. 2. Risk matrix – scheme, developed by MMS [2]


Letters refer to risk areas: little (L), medium (M), high (H). Using risk matrix method it is important to correctly identify probability category and expected loss category in relation to the scale of analysed hazards (events). One needs to bear in mind that subjective selection of the risk categories may be one of the weaknesses of the method. When using a three-tiers scale of risk, little risk equals to the area of activities taken within existing procedures, medium risk de facto equals to the area coming under crisis/risk management (in order to reduce risk to the acceptable level), high risk equals to the area, where immediate evacuation of people at risk is recommended. Probability values and potential effects in Matrix Method may be grouped conventionally, according to categories suggested in [10]. Another method for estimating the scope of probability of adverse events occurrence in Risk Matrix Method may be reference to Individual Risk of Death, estimated, among others, in [5] and [6]. It is also worth mentioning that Risk Matrix Method may be classified as one of „sharp edge” methods– here, change of the risk scale will be a result of a jump from one area of the risk factor value which influences risk, to the other. A similar method for risk analysis is, among the others, introduced by Kinney and Wiruth Risk Score Method [3].
5. Matrices in Crisis Management

Depending on the type of hazard, its scale and the state of its environment, some of the decisions may be made more often than the others. We are dealing here with some categorisation of decision problems, dependent on type of event (hazard). Given the correlation between the type of crisis event and its possible effects, they may be presented in a form of Hazard Matrix. Such a matrix makes it possible to estimate what type of adverse event may occur as a result of a specific hazard impact. Other type of matrix used for crisis management process is Safety Matrix [1]. It illustrates tasks assignment of all services, inspectors and guards (sometimes of other subjects engaged into process), depending on the type of adverse event. Usually, role division is used: there are prime, auxiliary and cooperation roles, and they are distributed within the needs.

Decision making process when in prevention and response phase (also preparation one) needs introducing two new, subsequent matrices.

The relationship between a type of adverse event and the most frequent decision problems (but those which appear after adverse event occurrence), may be presented in a shape of Decision Matrix (in the area of response) or, in short, R-decision Matrix. In the matrix, intensity of certain tasks (decision problems) occurrence is specified (quantitatively) in correlation with type of hazard from the point of view of the local government (crisis management team).

In the area of prevention activities which are taken before adverse event occurrence, Decision Matrix (in the area of prevention) may be defined or, in short, Z-decision Matrix. The matrix indicates the correlation between factors which characterise risk connected with specific hazard realisation and effect of specific decisions on reduction of „impact” of the hazard on the total risk value. Following columns Z-decision Matrix represent types of possible decisions and the rows – risk (or susceptibility) factors. For instance, one of the risk factors is number of people located in the threatened area. Public notification and alerting system modernisation, improving the conditions and possibility of evacuation, as well as increasing the degree of mass awareness may reduce influence of this element on total risk value. Any action to reduce the risk should be implemented based on the cost-effect analysis.
6. Associated Matrices Theory

Searching for the best i.e. the most effective decisions, as seen from the foregoing, may take place through using Hazard Matrix, Safety Matrix, R-decision Matrix, Z-decision Matrix and Risk Matrix. The above considerations lead to some correlations between specific matrices which are used for the needs of crisis management. At this point the argument may be made that coming to decision based on the use of matrices may be described with a use of a unique operation algorithm. The algorithm, which combines functionality of all the matrices and enables at least partial parameterisation and automatisation of decision making process, may be called Associated Matrices Theory (AMT). Picture 3 shows the scheme of procedure with the use of AMT. Firstly, hazard analysis is conducted, which leads to construction of a Hazard Matrix. The next stage of the process is risk analysis, which requires a full analysis of event trees (which describe possible scenarios after occurring the preliminary adverse event) and mistake trees (which describe how it was possible that the preliminary adverse event occurred). At this stage it is vital to identify the most important scenarios and correct quantitative analysis (estimating the value of probability and potential effects). Starting from qualitative and quantitative analysis of event and fault trees one comes to Risk Matrix. The last stage of AMT process is decision analysis and choosing the best option. The way of procedure (decision selection) depends on the stage of crisis management. In the prevention stage the most important role plays risk analysis implemented on the basis of Hazard Matrix, Risk Matrix and Z-decision Matrix (also with the use of Safety Matrix). In case, when the risk level (appointed with the use of Risk Matrix) is over the threshold of acceptable risk, it is necessary to reduce risk. Using Z-decision Matrix a decision maker can estimate what type of activities may reduce risk, taking into consideration risk susceptibility. In preparation and response phases the scheme of procedure is similar, but R-decision Matrix replaces Z-decision Matrix (while in preparation phase Z-decision Matrix may be used, providing that a period of time preceding adverse event occurrence is long enough). Using R-decision Matrix it is possible to estimate what type of decision making problems will need to be solved while the event occurs. Possible activities analysis should be conducted together with
division of tasks for individual subjects specified in Safety Matrix. For each decision making problem it is possible to estimate the influence of activities (or lack of activities) on risk. Due to the fact that specific activities influence the value of risk in some specific area, it is necessary to create a map of a threatened area.

Designated risk is part of a new matrix which may be described as Multidimensional Decision Matrix (MDM). Let MDM be a multidimensional matrix which is defined as follows:

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MDM = [D, T, Q, R, K, f]
\]

where:
- \( x, y \) – coordinates of the area covered by the planned action (usually it is the place where the source of danger is located),
- \( D = (D_{xy,0}, ..., D_{xy,n}) \) – set of possible actions,
- \( T = (T_{xy,0}, ..., T_{xy,n}) \) – corresponding to set \( D \) set of maximum times, when decision must be taken so that it will not lose its effectiveness,
- \( Q = (Q_{xy,0}, ..., Q_{xy,n}) \) – corresponding to set \( D \) set of resources necessary to make decision to start some actions,
- \( R = (R_{xy,0}, ..., R_{xy,n}) \) – corresponding to set \( D \) set of risk values in the specific area \((x, y)\) after starting some actions,
- \( K = (K_{xy,0}, ..., K_{xy,n}) \) – corresponding to set \( D \) set of criteria for choosing the best variation of activities,
- \( f \) – decision maker’s preference function,
- \( n \) – marker for a subsequent decision variation („0” refers to lack of action).

MDM Matrix is a composite dataset which contains the most important information, inevitable to make a decision during prevention, preparation and response phase. The matrix is built on the basis of the geospatial data of the area of interest (risk analysis of the area). The aim, which is to make the most effective decisions to ensure the safety of people and property, may be achieved through the spatial risk analysis taking into account, inter alia, the passage of time. When a crisis situation occurs, it usually comes to the accumulation of problems resulting in a constant time pressure. On the one hand, when time is passing, our information resource is growing bigger, on the other hand, however, time left for action taking is „shrinking”. Therefore it is vital to find optimal decision in the shortest possible time.
Fig. 3. Operational algorithm in Associated Matrices Theory
Rys. 3. Algorytm operacyjny w Teorii Matryc Powiązanych
In MDM matrix all data sets \((D, T, Q, R, K)\) include elements allocated according to geographical coordinates \((x, y)\) (or other spatial reference system – depending on the needs). The various alternatives (set \(D\)) are described by relevant risks (set \(R\)). Thanks to this approach the decision maker receives (within the available information) the full set of actions possible to be taken together with assignment of the highest risk areas, where the action is of the highest priority. It is important not to forget that set of activities \(D\) includes also action \(D_{xy,0}\), which represents „lack of action”. When considering only the action for \(D_{xy,0}\) for all combinations of coordinates \(x\) and \(y\), an initial, spatial distribution of risk (map of risk) for a specific moment of time and a specific area might be built.

Searching for priority actions takes place through the analysis of risk extremes in Multidimensional Decision Matrix. The first priority in all the areas, subject to the impact of a hazard, will be all the actions which reduce risk in a relatively best way. At the same time, the first priority for action will have areas of the highest risk value for the lack of action \((D_{xy,0})\). It follows that searching for priority actions (creating activities schedule) is performed, on the one hand, by searching for risk maximum for lack of action, and, from the other hand, (in already specified areas \((x_n, y_n)\)) by minimum searching in set \(R_{xy}\) e.g. such decisions which will reduce risk by the maximum amount.

The above presented concept is an open model of decision support based on risk analysis, which was described in a more detailed way in the monograph [8]. The implementation of a functioning Associated Matrices Theory requires continuous access to a current database of an area [9] where an event may occurs (or it is in progress). Because of the influence of various factors on risk values, such a database must be updated on a regular basis – also thanks to results of research on the social acceptance of risk.

References


Matrycowe metody zarządzania ryzykiem
– Teoria Matryc Powiązanych

Streszczenie

Bezpieczeństwo to jedna z podstawowych potrzeb człowieka. We współczesnym świecie zagrożenia naturalne przeplatują się z cywilizacyjnymi. Dążenie do poprawienia poziomu życia, wspieranie ludzi, z pomocą maszyn i technologii, doprowadziło do sytuacji, w których wzrost świadomości na temat bezpieczeństwa w niektórych obszarach naszego życia powoduje jego
ograniczenie w innych. Zjawisko to może być w prosty sposób wytłumaczone za pomocą transferu ryzyka, ale zrozumienie problemu nie oznacza jego rozwiązania.

Analiza ryzyka jest jednym z unikalnych narzędzi technicznych w dziedzinie zarządzania kryzysowego i ochrony ludności. Jedną z popularnych metod analizy ryzyka jest matryca ryzyka. Nowa koncepcja Teorii Matryc Powiązanych została opisana w artykule. Jest ona oparta na analizie matrycy ryzyka i matrycy bezpieczeństwa i może wspierać proces podejmowania decyzji w zakresie następujących faz zarządzania kryzysowego: zapobieganie, gotowość i reakcja. Metoda ta może być przydatna jako narzędzie matematyczne, które wykorzystuje wyniki analiz prawdopodobieństwa i konsekwencji w celu ustalenia najbardziej efektywnego wyboru dla decyzji, które powinny być wprowadzone w celu ochrony społeczeństwa, mienia i środowiska.

Przedstawione koncepcja jest otwartym modelem wspomagania decyzji na podstawie analizy ryzyka. Wprowadzenie funkcjonującej Teorii Matryc Powiązanych wymaga ciągłego dostępu do aktualnej bazy danych obszaru, gdzie zdarzenie może wystąpić (lub aktualnie występuje). Ze względu na wpływ różnych czynników na wartość ryzyka, takie bazy danych muszą być aktualizowane w regularnych odstępach czasu – m.in. poprzez wyniki badań społecznej akceptacji ryzyka.