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Modeling of Local Scour in Non-cohesive Soils Below Sills Using SSIIM Computer Code

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1. Introduction

As a result of river damming by a weir and/or dam structure with a reservoir, the flow regime changes and the balance of sediment transport is distorted. As a consequence, an increased energy of water and the discontinuity of sediment transport cause local scour in loose soils below the weir structure. In order to reduce the risk of undermining the weir foundation, various forms of bed protection are used, which weaken the process of local scour and shift it to a safe distance from the foundation. One of them are sills and horizontal (rigid or elastic) aprons.

Trials to predict the local scouring have lasted for more than 100 years. In the previous century, a lot of empirical equations with a narrow range of validity were formulated, describing basically the maximum depth of local scour. At the moment, apart from laboratory investigations on physical models, numerical modeling is often used.

The aim of this work is to check the usability the SSIIM code to simulate local scour development in noncohesive soils below rigid sills. Hitherto, the code was successfully (at least – partly) applied by [11, 1], as well as [7] for simulation of local scour around bridge piers and by [10] as well as [12] for mathematical modeling of water and sediment flow in a sand trap.

2. Materials and methods

2.1. Model description

For modeling of the local scour (Fig. 1) the SSIIM (A three dimensional numerical model for simulation of sediments movements in water intakes with multiblock options) computer code was used [13]. A two – dimensional scheme (including wall effects) was anlysed.

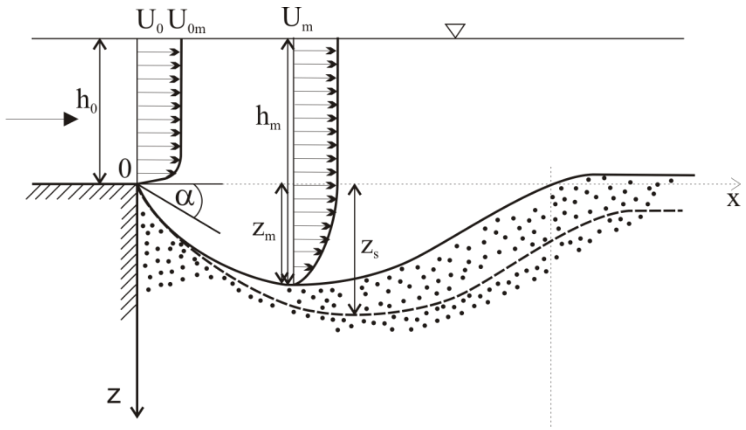

Fig. 1. Definition sketch showing a longitudinal profile of scour hole below a rigid sill: h_0, – water depth at the end of the sill, m, h_m – maximum water depth in scour hole, m, z_m – maximum depth of erosion, m, z_s – stabilized erosion depth, m, U_m – average horizontal velocity in the section of the greatest depth h_m, *m/s,* U_0 – average horizontal velocity at the end of the sill, m/s
Rys. 1. Szkic przedstawiający przekrój podłużny dolnego stanowiska budowli upustowej: h_0, – głębokość wody w przekroju końcowym umocnień, m, h_m – maksymalna głębokość wody w wyboju, m, z_m maksymalna głębokość erozji, m, z_s – ustabilizowana głębokość erozji, m, U_m – prędkość średnia w pionie o największej głębokości h_m, *m/s,* U_0 – prędkość średnia w pionie na końcu umocnienia, m/s

In the calculations of water flow two partial differential equations of the type: k-ε or k-ω were used, where ε means dissipation rate of turbulence kinetic energy k, while ω – specific dissipation rate of turbulence kinetic energy k.

Transport of suspended load was calculated using the convection–diffusion equation for sediment concentration c. To calculate the rate of bedload transport, the following formula by [15] was applied**:**

$$\frac{q_b}{d_{50}^{1.5}\sqrt{\delta\, g}} = b\left(\frac{\tau - \tau_c}{\tau_c}\right)^p d_{50}^{-0.3}\left(\frac{\delta \cdot g}{\nu^2}\right)^{-0.1} \tag{1}$$

where: q_b – bedload transport rate, m^3/sm, b – proportionality coefficient (0.0053 - 0.053),

τ – shear stress at the bottom, Pa,

τ_c – critical bed shear stress causing movement of sediment particles, Pa, p – exponent (1.8 – 2.1),

g – gravitational acceleration, m/s^2, d_{50} - characteristic value of the sediment particle diameter, m,

$\delta = (\rho_s - \rho)/\rho$ – relative density of the bed material, -,

ρ_s – sediment density, kg/m^3,

ρ – water density, kg/m^3,

ν – kinematic viscosity of water, m^2/s.

When in reality suspended transport is predominating, then the results obtained from the Eq. (1) are inaccurate. To determine the amount of sediment re-integrating the bedload with suspended load the following relation by van Rijn was used, describing the volume fraction [m^3/m^3] of the sediment located near the bottom:

$$c_{bed} = 0.015\frac{d_{50}^{0.7}}{a}\left[\frac{\tau - \tau_c}{\tau_c}\right]^{1.5}\left[\frac{\delta \cdot g}{\nu^2}\right]^{-0.1} \tag{2}$$

where:

c_{bed} – volumetric concentration of sediment near the bottom,

a – reference level equals to the roughness height, m.

Description of the local scour process is very complicated. In this study the impact of several parameters on the rate of sediment transport was investigated.

Sediment concentration affects the density of the mixture of water and the sediment changing the flow characteristics, for example forming density currents. This effect has been taken into account by adding the term $\rho_s g \frac{\partial c}{\partial z}$ to the Navier–Stokes equation, where c – volume concentration of sediment. Inclusion of this term brings about a change in direction of the bottom velocity.

In the SSIIM two special algorithms describing sediment movement on the sloped bottom are distinguished. The first one is associated with a reduction of critical shear stress using the Brooks formula [13], and the second is called “Sand slide”. The Brooks` coefficient reduces the effective value of the critical shear stress agitating sediment particles on the sloping bottom comparing to a horizontal one. The algorithm "Sand slide" causes the adaption of the simulated bottom slope to the angle specified by the user, e.g. the angle of soil repose under water.

2.2. Model identification

The area of research used to identify the above presented mathematical model was the Warta river below the Jeziorsko reservoir (Central Poland). The basic assumptions of the model were derived from [2] and special own research. The following geometry of the test area (Fig. 2) was assumed as: length 200 m, width 40 m, initial depth 2.61 m. The unit flow was steady and typical for a high flow q = 7.0 m^3/ms. It was assumed that there was no inflow of sediment to the movable bed. The initial 40 m of the bottom was protected by an apron made of stones of diameter 0.10 m, and the next 160 m was built of sand of diameter d_{50} = 0.5 mm.

The verification of the mathematical model was carried out in two stages. The first step was aimed to check the impact of the following factors on time evolution of the maximum scour depth: exponent p in Eq. (1), turbulent flow models k-ε and k-ω, reduction of the critical shear stress and "Sand slide" algorithm. The erosion time reached 16 hours under clear water scouring condition. The effective roughness of the bottom was used as $k_s = 3d_{90}$ and an additional term $\rho_s g \frac{\partial c}{\partial z}$ in Navier-Stokes equation was added.

To verify the simulation results the following, well-known Dutch empirical formula was used [5], [9]:

$$z_m = \frac{\left(U_{0m} - U_n\right)^{1.634} h_0^{\;0.24} t^{0.38}}{8.15\,\delta^{0.646}} \;\; [\mathrm{m}] \tag{3}$$

where: $U_{0m} = U_0 + 3\sigma_{u0}$ – maximum velocity at the end of the bed protection, m/s, U_n – non scouring velocity, m/s, t – time of erosion, h.

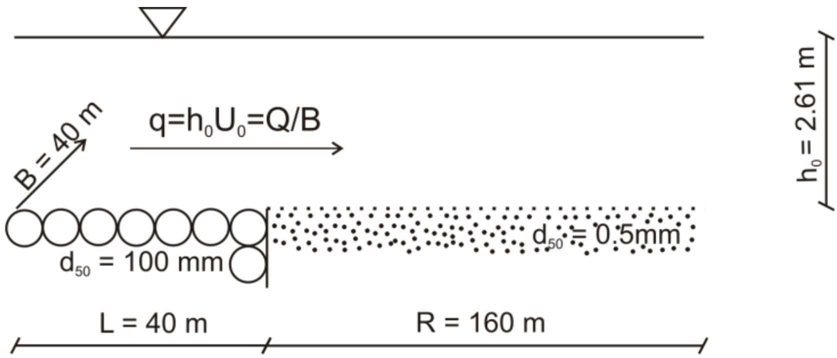

Fig. 2. Longitudinal profile of the simulated test area
Rys. 2. Profil podłużny symulowanego obszaru badawczego

In the second stage of the verification the influence of following factors was examined: standard algorithm TSC (Alg TSC) (transient sediment computation), algorithm referring to the numerical grid nodes at the bottom (in these points, the equation describing the concentration of sediment was transformed into the intensity of erosion (IntEr)), the movement of bedload and suspended load (Bl+Sl) simultaneously, as well as the movement of bedload (Bl) only. At this stage of simulation, the additional term in Navier-Stokes equation was taken into account, the turbulent flow model was model k-ω, the algorithm "Sand slide" was used for angle of repose 30 degrees, and exponent and constant in the formula by van Rijn had the default values (p = 2.1 and b = 0.053).

The input data for calculations and data on time evolution of scour holes were taken from the following works: [3, 4, 14]. In Table 1 the basic parameters of the laboratory tests are summarized.

Table 1. Basic parameters of laboratory tests used to calibrate the SSIIM model [8]
Tabela 1. Zestawienie podstawowych parametrów badań laboratoryjnych, wykorzystanych do kalibracji modelu SSIIM [8]

Exp.	Authors of laboratory tests	Duration of exp.	Water flow discharge	Water depth	Length of bed protection	Length of movable bed	Channel width	Sediment characteristic	
		t	Q	h_0	L	R	B	d_{50}	d_{90}
		h	dm^3/s	m	m	m	m	mm	mm
A	Breusers (1966, 67)	1.21	137	0.25	0.75	2.5	0.5	0.22	–
B	Breusers (1966, 67)	4	550	0.5	1.5	5	1.0	0.22	–
C	Breusers (1966, 67)	25	5085	1.5	4.5	15	3.0	0.22	-
D	Siwicki (2002)	8	73	0.165	0.5	4.5	1.07	0.44	0.7
E	Siwicki (2002)	8	73	0.165	0.5	4.5	1.07	0.29	0.58
F	Siwicki (2002)	8	73	0.165	0.5	4.5	1.07	1.1	2.4
G	Siwicki (2002)	8	122	0.215	0.5	4.5	1.07	0.44	0.7
H	Siwicki (2002)	8	122	0.215	0.5	4.5	1.07	0.29	0.58
I	Siwicki (2002)	8	122	0.215	0.5	4.5	1.07	1.1	2.4

Table 2 presents the of parameters used in this study. Designations S 1 – S 8 were used in the description of graphs in the section "Results".

Table 2. Specification of parameters in SSIIM mathematical model
Tabela 2. Parametry modelu matematycznego SSIIM

Set	k–ε	k–ω	Sand slide	Reduction of critical bed shear stress	Alg TSC	Alg Inter	Bl	Bl+ Sl
S 1	+	–	–	+	+	–	+	–
S 2	–	+	–	+	+	–	+	–
S 3	+	–	+	–	+	–	+	–
S 4	–	+	+	–	+	–	+	–
S 5	–	+	+	–	+	–	+	–
S 6	–	+	+	–	–	+	+	–
S 7	–	+	+	–	+	–	–	+
S 8	–	+	+	–	–	+	–	+

Notes: + ON, - OFF

Calculations have been carried out using PC with a central unit of the following configuration: Intel@CoreTM i7 860 and 4 GB RAM memory.

3. Results

As a result of the identification of the local scour model implemented in the SSIIM code it can be concluded that more correct results were obtained by means of using: additional term $\rho_s g \frac{\partial c}{\partial z}$ of the Navier - Stokes equation and k_s = $3d_{90}$, the constant in the equation for bedload discharge equal to the value proposed by van Rijn, i.e. b = 0.053.

Figure 3 shows geometric shape of the local scour below horizontal sill as in Fig. 2 after 16 hours of simulation for the parameters` set S 2 and the exponent p = 1.8. It is worth to note that the maximum scour holes have occurred at walls. In reality, the maximum scour depth has reached 4.6 m [2].

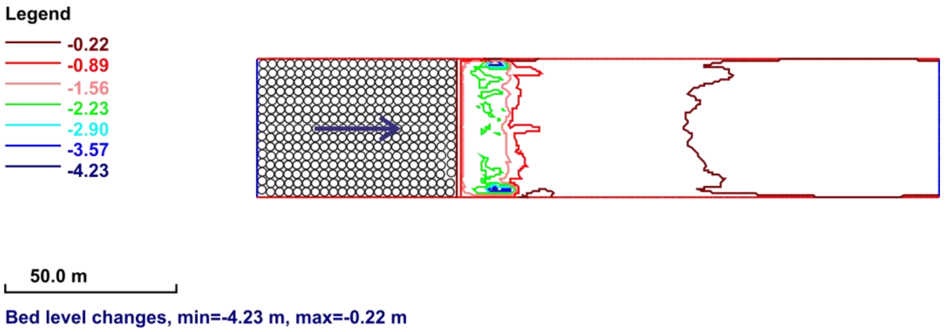

Fig. 3. Countour plan of the local scour below horizontal sill as in Fig. 2 after 16 hours of simulation (plan view)
Rys. 3. Kształt wyboju lokalnego poniżej poziomego progu z Rys. 2 po 16 godzinach symulacji (widok z góry)

Figure 4 shows evolution of the maximum scour depth in time for the scheme shown in Fig. 2 for different turbulent flow models, "Sand slide" algorithm and the reduction of critical shear stress as well as the exponent values in Eq. (1).

a) b)

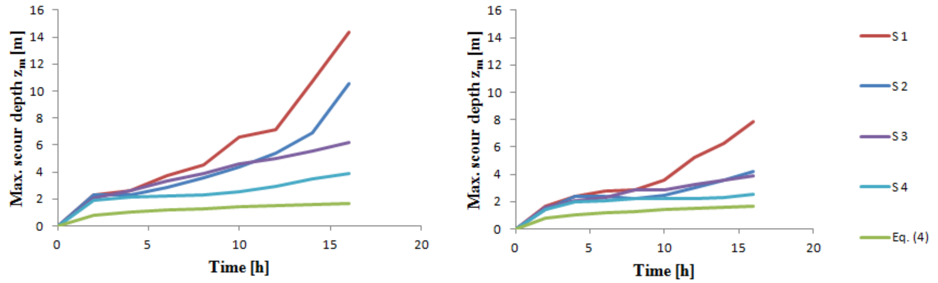

Fig. 4. Development of the maximum depth of local scour (as in Fig. 2) in time, depending on the criterion of sediment stability and the turbulent flow model for: a) the exponent p = 2.1 in Eq. (1) and b) for the exponent p = 1.8 in Eq. (1)
Rys. 4. Zmiany maksymalnej głębokości wyboju w czasie w zależności od kryterium stateczności gruntu i modelu turbulencji dla a) wykładnika potęgi 2.1 we wzorze van Rijna (5) b) dla wykładnika potęgi 1.8 we wzorze van Rijna (5)

On the basis of the first part of verification it was found that the application of reduction critical shear stress gave larger scour depths and less consistent with the empirical formula (4) than application of the algorithm "Sand slide", irrespective of the applied turbulence flow model. Using this configuration of the model, no stabilization of the local scour depth in time was achieved. The k-ε model gave greater maximum scour depths than the k-ω model for the same input data, which made it closer to the empirical Dutch formula Eq. (3). Statistical analysis was carried out on the basis of the Eq. (4). Relative mean square residual errors had values from ± 1.18 to ± 2.29.

Subsequent calculations – in the second part of the verification tests – were performed for laboratory experiments results provided by Siwicki [14]. Our study utilized his six experiments (D, E, F, G, H, I – conf. Table 1). Figure 5 presents a comparison of the geometry of the local scour hole calculated by the SSIIM model with the laboratory results (experiment H).

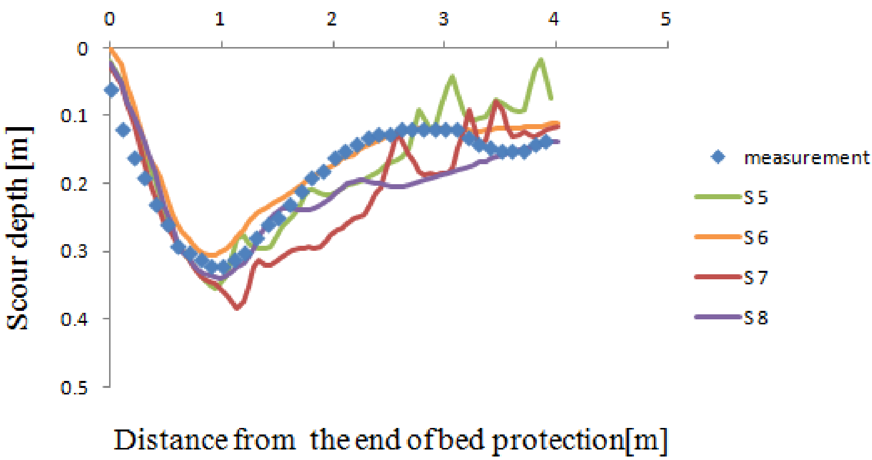

Fig. 5. Longitudinal profiles of local scour holes simulated by SSIIM code on the basis of [14] research**,** experiment H, after 8 h of erosion
Rys. 5. Geometria wyboju lokalnego obliczona modelem SSIIM na podstawie badań [14], doświadczenie H, czas trwania erozji 8 h

The calculated shapes of the scour hole using parameter`s set S 2, after 8 hours of erosion, show much better agreement with that measured in the experiment H than in the former ones.

As the goodness-of-fit criteria the following statistical measures were adopted after [6]:

• relative mean square residual error

$$MRE = \frac{100\%}{\overline{z_e}} \left(\frac{1}{n} \sum_{i=1}^{n} \left(z_{e,i} - z_{s,i} \right)^2 \right)^{\frac{1}{2}} \tag{4}$$

• the ratio of mean values

$$MWR = \frac{\overline{z_s}}{\overline{z_e}} \tag{5}$$

where: n – sample size, i.e. number of mesurement,

z_e – measured value of scour depth,

z_s – simulated value. The top bar means an average value of z.

Table 3 summarizes the statistical measures describing the compared of results from laboratory tests and the SSIIM code.

Table 3. Summary of statistical measures that describe the modeled local scour depths [8]

Tabela 3. Zestawienie miar statystycznych, opisujących modelowane rozmycie lokalne [8]

Exp.	Sample size *n*	Relative mean square residual error *MRE* (4) for parameter set				The ratio of mean values *MWR* (5) for parameter set			
		S 5	S 6	S 7	S 8	S 5	S 6	S 7	S 8
A	15	±0.57	±0.57	±0.58	±0.53	2.32	2.32	2.37	2.11
B	15	±0.73	±0.67	±0.75	±0.72	3.67	3.02	4.04	3.57
C	15	±0.71	±0.53	±0.78	±0.67	3.47	2.14	4.29	3.02
D	41	±0.31	-	±0.38	**±0.23**	**0.76**	-	1.62	1.31
E	41	-	-	**±0.22**	**±0.18**	-	-	1.28	**1.21**
F	41	±1.10	±1.31	±0.92	±1.77	0.48	0.43	0.52	0.36
G	41	**±0.07**	**±0.17**	**±0.18**	**±0.18**	**0.94**	**0.85**	**1.22**	**1.22**
H	41	**±0.07**	**±0.18**	**±0.10**	**±0.05**	**0.93**	**0.85**	**1.11**	**1.06**
I	41	-	-	±1.67	-	-	-	0.38	-

Note: the goodness-of-fit model estimates: ***Bold*** *font – satisfactory, Plain point font – unsatisfactory, – unstable numerical solution*

The relative mean square residual error has ranged from ± 0.05 to ± 1.77. The ratio of mean values has shown also large variability 0.36–3.47 comparing with the best 1.0. The results obtained using the SSIIM code can be defined as good, pretty good and unsatisfactory. The results obtained for Breusers` experiments A, B i C have occurred wholly unsatisfactory. The cause of this discrepancy was probably an inadequate description of the sediment transport. More consistent results were obtained in simulations of the experiments by Siwicki, apart from the largest diameter of the sediment particles d_{50} = 1.1 mm and the largest geometric standard deviation $((d_{84}/d_{16})^{0.5})$, regardless of flow rate (experiments F and I). The best results were obtained for experiments G and H, where the bed material was relatively uniform, and the flow discharge was greater than in D. The most numerically unstable was experiment I, in which the bed material was non–uniform and the flow discharge had a greater value.

4. Conclusions

As a result of identification of the clear – water local scour below a horizontal rigid sill implemented in the SSIIM code for the erosion time 8 h it can be concluded that the more correct results are obtained using the additional term of the Navier–Stokes equation $\rho_s g \dfrac{\partial c}{\partial z}$ and effective bed roughness k_s = 3 d_{90}.

On the basis of the first part of the verification it was found that the use of critical shear stress reduction gave a greater depth of scour than using the algorithm "Sand slide", irrespective of the applied turbulence model. Model k-ε gave greater maximum scour depths than k-ω model for the same input data.

The simulation results obtained for Breusers` laboratory experiments have proved unsatisfactory. Unsatisfied results were obtained also for the largest diameter of sediment $d_{50}$ = 1.1 mm and its largest geometric standard deviation regardless of the flow rate in the Siwicki`s experiments F and I. The best results were obtained for experiment H, where the sediment was relatively uniform, and the flow was greater than in experiment D. The simulation of experiment I turned to be the most numerically unstable. In this experiment the sediment was mostly non-uniform, and the flow was relatively high.

The usefulness of the SSIIM model for the calculation of the local scour in non-cohesive soils below sills is promising but still problematic and therefore more research is needed, especially on sediment transport within the local scour hole.

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Modelowanie rozmyć miejscowych w gruntach niespoistych poniżej progów piętrzących za pomocą programu SSIIM

Streszczenie

Celem pracy jest sprawdzenie możliwości wykorzystania programu SSIIM (A three dimensional numerical model for simulation of sediments movements in water intakes with multiblock options) do modelowania rozmyć miejscowych, w gruntach niespoistych poniżej budowli piętrzących, progów lub innych umocnień dna rzeki. W artykule przedstawiono identyfikację wybranych parametrów programu SSIIM, istotnych dla obliczeń wyboju lokalnego w czasie. Geometrie wyboju uzyskane z obliczeń programem SSIIM porównano z geometriami wyboju uzyskanymi na modelach fizycznych w laboratoriach oraz obliczonymi za pomocą wzorów empirycznych.